KEY CONCEPTS AND FORMULAE

1. Real Numbers

   1. Euclid's Division Algorithm : Given positive integers `a' and `b', there exists unique integers q and r such that a=bq+r, where $0 \leq r < b$

      a Dividend   q quotient

      b Divisor   r remainder

2. Polynomials

   1. For a quadratic polynomial ax^2+bx+c,

      $$\alpha+\beta=\frac{-b}{a}, \quad \alpha\beta=\frac{c}{a}$$

      where α, β are roots/zeroes of polynomial.

   2. For a cubic polynomial $ax^3 + bx^2 + cx + d$

      $$\alpha+\beta+\gamma=\frac{-b}{a}, \quad \alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}, \quad \alpha\beta\gamma=\frac{-d}{a}$$

      where α, β, γ are roots/zeroes of polynomial.

   3. For any polynomial p(x) & g(x)

      p(x) = g(x) q(x) + r(x) when r(x) = 0 or deg. r(x) < deg g(x)

3. Pair of Linear Equations in two variables

   $a_1x + b_1y + c_1=0$

   $a_2x + b_2y + c_2=0$

   Nature of Roots/Zeroes/Solutions

a) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$

1) System has unique solution

2) Consistent

3) Graph is two intersecting lines.

b) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$

1) System has no soultion.

2) Inconsistent

3) Graphs are parallel lines.

c) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$

1) System has infinite solution

2) Consistent

3) Graphs are coincident lines.

4. Quadratic Equations

i) Standard form : $ax^2+bx+c = 0$

ii) Discriminant, $D=b^2-4ac$

iii) General roots (solutions) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (Quadratic formula)

iv) Nature of roots

a) If $b^2-4ac > 0$ Real distinct zeros,

b) If $b^2-4ac = 0$ Real equal roots,

c) If $b^2-4ac < 0$ No real roots,

d) If $b^2-4ac \geq 0$ Real roots

5. Arithmetic Progression (A.P.)

i) Standard form a, a+d, a+2d .... a+(n-1) d

where a= first term, d=common difference, t_n=a+(n-1)d

ii) Sum of `n' terms :

6. Triangles

i) Basic proportionality Theorem (Thales Theorem)

a) In a triangle if DE||BC then $\frac{AD}{BD} = \frac{AE}{EC}$

b) If $\frac{AD}{BD} = \frac{AE}{EC}$ then DE||BC

ii) Pythagoras Theorem

iii) Converse of Pythagoras theorem $S_n = \frac{n}{2}[2a + (n-1)d]$

iv) Ratio of areas of two similar triangles

If $\Delta ABC \sim \Delta PQR$ then $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{PR}\right)^2$

7. Coordinate Geometry

i) Distance Formula : $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

ii) Section Formulae : $x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$, $y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$

iii) Mid-point formula : $x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

iv) Area of $\Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

where (x_1, y_1), (x_2, y_2), (x_3, y_3) are vertices of triangle.

8. Introduction to Trigonometry.

i) In a right triangle ABC, right angled at B,

$$\text{SinA} = \frac{\text{Side opp. to angle A}}{\text{hypotenuse}}, \ \text{Cos A} = \frac{\text{Side adjacent to angle A}}{\text{hypotenuse}},$$

$$\tan A = \frac{\text{SinA}}{\text{CosA}} = \frac{\text{Side opp. of A}}{\text{Side adjacent to A}}$$

ii) $\text{Cosec A} = \frac{1}{\text{Sin A}}, \ \text{SecA} = \frac{1}{\text{CosA}}, \text{CotA} = \frac{1}{\tan A}$

iii) Sin (90° -A) = CosA, Cos (90°-A) = SinA, tan (90°-A) = Cot A

Cot (90°-A) = tan A, Sec (90°-A) = CosecA, Cosec(90°-A) = SecA

iv) $\text{Sin}^2\text{A} + \text{Cos}^2\text{A}=1$, $\text{Sec}^2\text{A}-\tan^2\text{A}=1$, $\text{Cosec}^2\text{A}-\text{Cot}^2\text{A}=1$

10. Circles $\rightarrow$

i) Thm - 1 $\rightarrow$ The tangent to a circle is perpendicular to the radius through the point of contact.

ii) Thm - 2 The lengths of two tangents from an external point to the circle are equal.

Chapter 1

Minimum Level of Learning (MLL)

Unit- I (Real Numbers)

Key Points :

1. Euclid's division Lemma :- For any two given positive integers a and b, there exists unique integers q and r satisfying.

   $a = bq+r, o \leq r < b$

2. Fundamental theorem of arthiemetic

   Every composite number can be expressed as a product of primes, and this factorisation is unique apart from the order in which the prime factor occurs.

3. Let x be a rational number whose decimal expansion terminates then x can be expressed in the form $\frac{p}{q}$. where p and q are co-prime and the prime factorisation of q is of the form $2^n.5^m$. where n, m are non negative integers.

4. For any two given numbers p and q

   H.C.F. (p, q) x L.C.M. (p, q) = p x q

Chapter - 2

Polynomials

Key Points

1. If p(x) is a polynomial in x, the highest power of x in p(x) is called the degree of the polynomial p(x). For exp. 4x+2 is a polynomial in the variable x of degree 1, $2y^2-3y+4$ is a polynomial in the variable y of degree 2.

2. Zero of a polynomial - A real constant K is said to be a zero of a polynomial p(x) in x, if p(k) = 0

   for exp. $p(x) = x^2+x-12$ gives $p(3) = 3^2+3-12=0$

   and $p(-4) = (-4)^2 + (-4) - 12 = 0$. Thus α,3β and -4 are two zeroes of the polynomials p(x)

3. Relation between zeroes and co-efficient of a polynomial.

   Let $p(x) = ax^2+bx+c$, $a \neq 0$ and having Zeroes as , then

   Sum of the Zeroes = $\alpha, + \beta = \dfrac{-(\text{co}-\text{efficient of } x)}{(\text{co}-\text{efficient of } x^2)} = \dfrac{-b}{a}$

   Product of the Zeroes = $\alpha\beta = \dfrac{\text{constant term}}{(\text{co}-\text{efficient of } x^2)} = \dfrac{c}{a}$

Chapter - 3

Minimum Level of Learning

Unit-III

Pair of Linear Equations in two variables

Key Points

General Form of a Linear equation.

$ax + by + C = 0$, where a, b, c are real numbers and x,y are variables.

Solution of a pair of linear equations in two variables

A pair of values of x and y, satisfying each one of the questions, is called a solution of the linear equations in two variables.

There are three possibilities -

i) the two lines will intersect in one point.

ii) the two lines will not intersect i.e. they are parallel.

iii) the two lines will be coincident.

Conditions for consistent/inconsistent

A pair of linear equations in two variables, which has a solution, is called consistent and a pair of linear equation in two variables, which has no solutions is called inconsistent.

If a pair of linear equation in two variables is given by

a,x + b_1y+c = 0 and a_2x + b_2y +c_2 =0 then, they have

i) unique solution (consistent), if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Constructions

1. To divide a line Segment in a given ratio.

2. Construction of Similar Triangles.

3. Construction of pair of tangents from an external point.

Areas related to circle

1. Circumference of a circle = $2\Pi r$,

2. Area of circle = Π r^2

3. Length of an arc of a sector $= \frac{\vartheta}{360^\circ} \text{ x } 2\Pi r$

4. Area of a sector $= \frac{\vartheta}{360^\circ} \text{ x } \Pi r^2$

5. Area of segment of a circle

= Area of the corresponding sector - Area of triangle.

Surface Areas and Volumes

1. Volumes of frustum of a cone = $\frac{1}{3} \text{ x } \Pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

2. Curved surface area of frustum = $\Pi \ell \left(r_1 + r_2\right)$ where $l = \sqrt{h^2 + \left(r_1 - r_2\right)^2}$

3. Total surface area of the frustum = $\Pi l\left(r_1 + r_2\right) + \Pi \left(r_1^2 + r_2^2\right)$

Statistics

1. Mean $=\bar{x}=\frac{\Sigma fixi}{\Sigma fi}$ (Direct method)

$=a+\frac{\Sigma fidi}{\Sigma fi}$ (Assumed Mean method)

$=a+\left(\frac{\Sigma fidi}{\Sigma fi}\right)x\,h$ (Step deviation method)

2. Mode $=l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)x\,h$

3. Median $=l+\left(\frac{\frac{n}{2}-cf}{f}\right)x\,h$

Probability

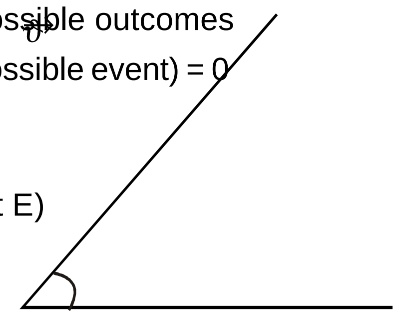
1. $P(E)=\frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes}}$

2. p(sure event) = 1, p (impossible event) = 0

3. $O \le P(E) \le 1$

4. $P(E)+P(\bar{E})=1$ ($\bar{E} \rightarrow$ not E)

Heights and Distances



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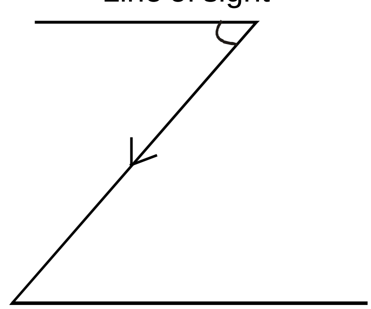

Line of sight

ϑ

Line of sight

Object

ϑ Angle of elevatin

Angle of depression

Chapter-1

Section A

Very Short Answer type questions (each of Mark - 1)

Q.1 State whether is rational or irrational number.

Q.2 "π is a rational number", this is true statement or false.

Q.3 Express 156 as a product of prime factors.

Q.4 Fill in the blank.

H.C.F. (p, q) x __________= p x q

Q.5 Convert into decimal expansion.

Q.6 Can we have any $n \in N$ where 6^n ends with digit zero.

Section B

$\sqrt{2}$ $\frac{5}{8}$

Short Answer Type Questions (each of Marks 2)

Q.1 Use Euclid division alogrithim to find H.C.F. of

i) 56, 814 ii) 6265 and 76254

Q.2 Find HCF and LCM of following using fundamental theorem of arithmetic method (Prime factorisation)

i) 6, 72, 150 ii) 26, 91

Q.3 Examine whether the following numbers are terminating or non terminating (without actual division)

i) $\frac{35}{51}$ ii) $\frac{63}{90}$ iii) $\frac{513}{2^2 5^7 7^3}$

Unit 1

Real Number

Example 1. Use Euclid's division alogrithim to find the H.C.F. of 867 and 255.

So. : Since 867> 255

$\therefore$ 867 = 255 x 3 + 102

Now 102 is remainder, 255 can be written as

255 = 102 x 2+51

Again 51 is remainder, 102 can be written as

102 = 51 x 2+0

Now remainder is zero $\therefore \frac{15}{1600}$

last divisor will be the H.C.F.

H.C.F. (867, 255) = 51

Example 2. Wihout actually performing the long division, state whether the will have a terminating decimal or Non-terminating repeating decimal expansion.

Sol. : $\frac{15}{1600} = \frac{3 \times \cancel{5}}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times \cancel{5}} = \frac{3}{2^6 \times 5^1}$

Since demoninator is of the form $2^n.5^m$, so its decimal expansion will be terminating type.

Q.4 Prove that 5 + $\sqrt{3}$ is irrational.

Q.5 Prove that $3\sqrt{5}$ is irrational

Q.6 Show that $3\sqrt{6}$ is irrational number.

Q.7 Given that HCF (306, 657) = 9, Find LCM (306, 657)

Section C

Short Answer Type Question (Each of Marks 3)

Q.1 Show that any positive odd integer is of the form 4q+1 or 4q+3, where q is some integer.

Q.2 Explain why 7 x 11 x 13 +13 is a composite number.

Q.3 Prove that no number of type 4k + 2 be a perfect square.

Q.4 Prove that $\sqrt{3}$ is an irrational number.

Q.5 Use Euclid's division alogrithim to show that square of any integer is either of form 3m or 3m+1 for some integer m.

Answers

Section A

(1) Irrational (2) False (3) 2 x 2 x 3 x 13 (4) L.C.M. (p,q) (5) 0.625 (6) No

Section B

(1) (i) 2 (ii) 179 (2) (i) 6, 360 (ii) 13, 182 (3) (i) Non terminating (ii) terminating (iii) Non terminating (7) 22338

Section A

1. $\sqrt{2}$ is irrational

   Let us assume that $\sqrt{2}$ is irrational

   20, we can find integers r and s ($\neq 0$) such

   Suppose r and s have a commonfactor other than ℓ. Then, we divide by the

   commonfactor (r and s) to get $\sqrt{2} = \frac{a}{b}$ where a & b are comprime number

   So, $b\sqrt{2} = a$

   Squaring both sides

   $2b^2 = a^2$

   This shows that 2 divides a^2 $\sqrt{2} = \frac{r}{S}$ $\pi = \frac{22}{7} = 3.14$

   we can write 1a = 2c for some integer c substituting for a, we get $2b^2 = 4c^2$ that

   is $b^2 = 2c^2$ $b^2 = \frac{4c^2}{2} = 2c^2$

   This shows that 2 also divides b^2, So a divides b

   $\therefore$ a & b have 2 as a commonfactor.

   But this contradicts the fact that a & b have no common factor other than 1.

   This contradiction shows that our assumption is wrong. is irrational.

2. π is irrational because is rewritten in the form of

3. 156 $\Rightarrow$ 2 x 2 x 3 x13

4. H.C.F. (p, q) x L.C.M. = p x q

5.

```
8)50(.625
  48
 ------
  20
  16
 -----
  40
  40
 -----
   x
 -----
```

Section B

2. i) 6, 72, 120

6 = 2x3, 72 = 2 x 2 x 2 x 3 x 3

120 = 2 x 2 x 2 x3 x 5 $\frac{5}{8} = 0.625$ $5+\sqrt{3} = \frac{a}{b}$

H.C.F. (6, 72, 120) = 2 x 3=6

LCM (6,72,120) = 2x2x2x3x2x5=360

ii) 26, 91

26 = 13x2

91 = 13 x 7

H.C.F. = 13

LCM = 2 x 13 x 7= 182

4. 5 + $\sqrt{3}$ is irrational

Let us assume to the contrary 5+$\sqrt{3}$ is rational. We can find coprime a & b

(b$\neq$0) such that

$\therefore 5 - \frac{a}{b} = -\sqrt{3}$

Rearranging this equation $-\sqrt{3} = \frac{5-a}{b} = \frac{5b-a}{b}$ where a & b are intergers we

get $\frac{5b-a}{b}$ is rational, ans so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ as irrational.

This contradiction shows that our assumption is incorrect 5+ $\sqrt{3}$ is irrational.

5. $3\sqrt{5}$ is irrational

Let us assume to the contrary that $3\sqrt{5}$ is irrational.

That is, we can find coprime a&b (b $\neq$ 0) such that . Rearranging we get $\sqrt{5} = \frac{a}{3b}$

Since 3, a & b are integers $\frac{a}{3b}$ is rational. So, is rational

But this contradicts the fact that $\sqrt{5}$ is irrational

So, we conclude that 3 $\sqrt{5}$ is irrational.

6. $3\sqrt{6}$ is irrational number. $3\sqrt{5} = \frac{a}{b}$ $3\sqrt{6} = \frac{a}{b}$

Let us assume to the contrary that $3\sqrt{6}$ is rational.

That is, we can find comprime a&b (b $\neq$ 0) such that

Rearranging $\sqrt{6} = \frac{a}{3b}$

Since 3, a &b are integers, $\frac{a}{3b}$ is rational.

So is also rational. But this contradicts the fact that $\sqrt{6}$ is irrational. So, we conclude 3 $\sqrt{6}$ is irrational.

7. $\text{LCM} = \frac{1^{st}\text{ No x II}^{nd}\text{ No}}{\text{HCF}}$

$$= \frac{306 \text{ x } 657}{9} = 22338$$

Section -C

1. Let us start with taking a where a is postive integers. We apply the division algorithm with a & b = 4

Since 0 < r <4, the possible remainder are 0,1,2 and 3

That is a, can be 4q or 4q +1 or 4q +2 or 4q +3

However since a is odd, a can not be 4q or 4q+2 (because they are divisible by 2)

$\therefore$ Any odd integers is the form of 4q +1 or 4q +3

2. 7 x 11 x 13 + 13

13 (7 x 11 + 1)

13 (77+1)

13 x 78 = 1014, Yes it is composite number.

Section B

Q.1 i) 56, 814

814= 56x14+30

56 = 30x1+26

30 = 26x1+4

26 = 4 x 6+2

4 = 2 x 2+0

ii) 76254 & 6265

76254 = 6265x12+1074

6265 = 1074 x 5 + 895

1074 = 895 x1+179

895 = 179 x 5 + 0

H.C.F. = 179

Section A

6. No, Because it will not end with digit zero. If the prime factorisation of 6^n contain 5. But the prime factorisation of 6^n only contain 2 & 3.

$6^n = (2x3)^n = 2^n \times 3^n$.

(1) - Non terminating

$35 = \frac{35}{3 \text{x} 17}$ Non terminating

(2) $\frac{63}{90} = \frac{7}{10} = \frac{7}{5\text{x}2}$ terminating

(3) $\frac{513}{2^2 5^7 7^3}$ $\Rightarrow$ Non terminating

Section C

5. Let a be any positive integer, and b=3 $\frac{33}{51}$

a=3q+r

0 < r <b

r=0, 1 or 2		$(3q+1)^2 = (9q^2+1+6q)$
a= 3q		$= 9q^2 + 6q + 1$
a=3q+1		$= 3(3q^2+2q)+1$
a=3q+2		=3m+1
$(3q)^2 = 9q^2$		
$3(3q^2)$	$(3q+2)^2$	$= 9q^2+4+12q$
= Let $3q^2$ is m		$= 9q^2+12q+3+1$
= 3m		$=3(3q^2+4q+1)+1$
		= 3m+1