Part -A

1. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Solution: 12 = a + 2d

106 = a + 49d

So, 106-12 = 47d

Or, 94 = 47d

Or, d = 2

Hence, a = 8

And, n_{29} = 8 + 28x2 = 64

2. If the 3rd and the 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?

Solution: -8 = a + 8d

4 = a + 2d

Or, -8 – 4 = 6d

Or, -12 = 6d

Or, d = -2

Hence, a = -8 + 16 = 8

0 = 8 + -2(n-1)

Or, 8 = 2(n-1)

Or, n-1 = 4

Or, n = 5

3. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Solution: $n_7 = a + 6d$

And, $n_{10} = a + 9d$

Or, a + 9d – a – 6d = 7

Or, 3d = 7

Or, d = 7/3

4. Which term of the AP: 3. 15, 27, 39, … will be 132 more than its 54th term?

Solution: d = 12,

132/12 = 11

So, 54 + 11 = 65th term will be 132 more than the 54th term.

5. How many three digit numbers are divisible by 7?

Solution: Smallest three digit number divisible by 7 is 105

Greatest three digit number divisible by 7 is 994

Number of terms

= {(last term – first term)/common difference }+1

= {(994-105)/7}+1

= (889/7)+1=127+1=128

6. How many multiples of 4 lie between 10 and 250?

Solution: Smallest number divisible by 4 after 10 is 12,

The greatest number below 250 which is divisible by 4 is 248

Number of terms: {(248-12)/4}+1

{236/4}+1 = 59+1 = 60

7. For what value of n, are the nth terms of two APs: 63, 65, 67,… and 3, 10, 17,… equal?

Solution: In the first AP a = 63 and d = 2

In the second AP a = 3 and d = 7

As per question,

63+2(n-1) = 2+ 7(n-1)

Or, 61 = 5 (n-1)

Or, n-1 = 61/5

As the result is not an integer so there wont be a term with equal values for both APs.

8. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Solution: As the 7th term exceeds the 5th term by 12, so the 5th term will exceed the 3rd term by 12 as well

So, $n_3 = 16$

$n_5 = 28$

$n_7 = 40$

n_4 or n_6 can be calculated by taking average of the preceding and next term

So, $n_4 = (28+16)/2 = 22$

This gives the d = 6

AP: 4, 10, 16, 22, 28, 34, 40, 46, ……..

9. Find the 20th term from the last term of the AP: 3, 8, 13, ……, 253.

Solution: a = 3, d = 5

253 = 3 + 5(n-1)

Or, 5(n-1) = 250

Or, n-1 = 50

Or, n = 51

So, the 20th term from the last term = 51 – 19 = 32nd term

Now, n_{32} = 3 + 5x31 = 158

10. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and the 10th terms is 44. Find the first three terms of the AP.

Solution: a + 3d + a + 7d = 24

Or, 2a + 10d = 24

Similarly, 2a + 14d = 44

So, 44 – 24 = 4d

Or, d = 5

2a + 10x5 = 24

Or, a + 25 = 12

Or, a = -13

So, first three terms of AP: -13, -8, -3,

11. Subba Rao started work in 1995 at an annual salary of Rs. 5000 and received an increment of Rs. 200 each year. In which year did his income reached Rs. 7000.?

Solution: 7000 = 5000 + 200(n-1)

Or, 200(n-1) = 2000

Or, n-1 = 10

Or, n = 11

12. Ramkali saved Rs. 5 in the first week of a year and then increased her weekly savings by Rs. 1.75. If in the nth week, her savings become Rs. 20.75, find n.

Solution: 20.75 = 5 + 1.75(n-1)

Or, 1.75(n-1) = 15.75

Or, n-1 = 9

Or, n = 10

Part -B

1. Find the sum of the following APs:

(i) 2, 7, 12, ……, to 10 terms

Solution: $S = \frac{n}{2}(2a + d(n-1))$

Here, a = 2, d = 5 and n = 10

Sum $= \frac{10}{2}(2+2+45)$

$= 5 \times 49 = 245$

(ii) -37, -33, -29, …… to 12 terms

Solution: Here, a = -37, d = 4 and n = 12

$S = \frac{12}{2}(2 \times -37 + 4 \times 11) = 6 \times -30 = -180$

(iii) 0.6, 1.7, 2.8, …… to 100 terms

Solution: Here a = 0.6, d = 1.1 and n = 100

$S = \frac{100}{2}(0.6 + 1.1 \times 99) = 50 \times 109.5 = 5475$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots$ to 11 terms

Solution: Here a = $\frac{1}{15}$, $d = \frac{1}{60}$ and n = 11

$S = \frac{11}{2}\left(\frac{2}{15} + \frac{1}{60} \times 10\right) = \frac{11}{2}\left(\frac{4+5}{30}\right) = \frac{11}{2} \times \frac{3}{10} = \frac{33}{20}$

2. Find the sums given below:

(i) 7+10.5+14+…..+84

Solution: Here a = 7, d = 3.5

So, $84 = 7 + 3.5(n-1)$

$\Rightarrow 3.5(n-1) = 77$

$\Rightarrow n - 1 = 77 \div 3.5 = 22$

$\Rightarrow n = 23$

Now, $S = \frac{23}{2}(7 + 84) = 1046.5$

(ii) 34+32+30+….+10

Solution: Here a = 34, d = -2

So, $10 = 34 - 2(n-1)$

$\Rightarrow 2(n-1) = 24$

$\Rightarrow n - 1 = 12$

$\Rightarrow n = 13$

So, $S = \frac{13}{2}(34 + 10) = 286$

(iii) -5+(-8)+(-11)+…….+(-230)

Solution: Here a = -5, d = -3

So, $-230 = -5 - 3(n-1)$

$\Rightarrow 3(n-1) = -225$

$\Rightarrow n - 1 = -75$

$\Rightarrow n = 76$

Now, $S = \frac{76}{2}(-5 + -230) = 38 \times -235 = -8930$

3. In an AP:

(i) Given a = 5, d = 3, a_n = 50, find n and S_n.

Solution: $50 = 5 + 3(n-1)$

$\Rightarrow 3(n-1) = 45$

$\Rightarrow n - 1 = 15$

$\Rightarrow n = 16$

Now, $S_n = \frac{16}{2}(5 + 50) = 440$

(ii) Given a = 7, a_{13} = 35, find d and S_{13}.

Solution: $35 = 7 + d(13-1)$

$\Rightarrow 12d = 28$

$\Rightarrow d = 28 \div 12 = 2.33$

$S_{13} = \frac{13}{2}(7+35) = 273$

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}.

Solution: $37 = a + 3(12-1) = a + 33$

$\Rightarrow a = 37 - 33 = 4$

$S_{12} = \frac{12}{2}(4+37) = 246$

(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

Solution: $S_{10} = 125 = \frac{10}{2}(2a + 9d)$

$\Rightarrow 2a + 9d = 25 \ldots\ldots\ldots\ldots (i)$

$a_3 = a + 2d = 15 \ldots\ldots\ldots\ldots (ii)$

Subtracting equation (ii) from (i) we get

$a + 7d = 10$ (the 8^{th} term)

Subtracting 3^{rd} term from 8^{th} term we get

$(a + 7d) - (a + 2d) = 10 - 15 = -5$

$\Rightarrow 5d = -5$

$\Rightarrow d = -1$

So, a = 17

$a_{10} = 17 - 9 = 8$

(v) Given $d = 5$, $S_9 = 75$, find a and a_9.

Solution: $S_9 = \frac{9}{2}(2a + d(9-1))$

$\Rightarrow 75 = 4.5(2a + 8d)$

$\Rightarrow 2a + 8d = \frac{75}{4.5}$

$\Rightarrow 2a = \frac{150}{9} - 40 = \frac{150 - 360}{9} = \frac{210}{9} = \frac{70}{3}$

$\Rightarrow a = \frac{35}{3}$

Now, $a_9 = \frac{35}{3} + 40 = \frac{155}{3}$

(vi) Given a = 2, d = 8, S_n = 90, find n and a_n.

Solution: $S_n = 90 = \frac{n}{2}(2a + d(n-1))$

$\Rightarrow 180 = n(4 + 8n - 8) = n(8n - 4) = 8n^2 - 4n$

$\Rightarrow 8n^2 - 4n - 180 = 0$

$\Rightarrow 2n^2 - n - 45 = 0$

$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$

$\Rightarrow 2n(n-5) + 9(n-5) = 0$

$\Rightarrow (2n+9)(n-5) = 0$

So, n = 5 (this is the positive integer)

$\therefore a_5 = 2 + 8 \times 4 = 34$

(vii) Given a = 8, a_n = 62, S_n = 210, find n and d.

Solution: $S_n = 210 = \frac{n}{2}(8 + 62)$

$\Rightarrow 35n = 210$

$\Rightarrow n = 210 \div 35 = 6$

Now, $62 = 8 + 5d$

$\Rightarrow 5d = 62 - 8 = 54$

$\Rightarrow d = 54 \div 5 = 10.8$

(viii) Given a_n = 4, d = 2, S_n = -14, find n and a.

Solution: $a_n = 4 = a + 2(n-1)$

$\Rightarrow a = 4 - 2n + 2 = 6 - 2n$

$S_n = -14 = \frac{n}{2}(2a + 2(n-1))$

$\Rightarrow -28 = n(6 - 2n + 2n - 1) = 5n$

$\Rightarrow n = -28 \div 5 = -5.6$

So, $a = 6 - 2n = 6 + 6.2 = 12.2$

(ix) Given a = 3, n = 8, S = 192, find d.

Solution: $S_8 = \frac{8}{2}(6 + d(8-1) = 4(6 + 7d)$

$\Rightarrow 192 \div 4 = 48 = 6 + 7d$

$\Rightarrow 7d = 42$

$\Rightarrow d = 6$

(x) Given n_l = 28, S = 144, and there are total 9 terms. Find a.

Solution: $S_l = 144 = \frac{9}{2}(a + 28)$

$\Rightarrow 144 \times \frac{2}{9} = 32 = a + 28$

$\Rightarrow a = 32 - 28 = 4$

4. How many terms of the AP: 9, 17, 25,…. Must be taken to give a sum of 636?

Solution: a = 9, d = 8,

$S_n = 636 = \frac{n}{2}(18 + 8(n-1))$

$\Rightarrow 1272 = 10n + 8n^2$

$\Rightarrow 4n^2 + 5n - 636 = 0$

Root = $\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{5 \pm \sqrt{25 + 10176}}{8} = \frac{5 \pm \sqrt{10201}}{8} = \frac{5 \pm 101}{8}$

$\alpha = \frac{106}{8} = \frac{53}{4}$ $\beta = \frac{-96}{8} = -12$

As twelve is the integer so number of terms will be 12

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Solution: $S_n = 400 = \frac{n}{2}(5+45)$

$\Rightarrow n = \frac{400}{25} = 16$

Now, $a_{16} = 45 = 5 + d \times 15$

$\Rightarrow 15d = 40$

$\Rightarrow d = \frac{8}{3}$

6. The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is the sum?

Solution: $a_n = 350 = 17 + 9(n-1)$

$\Rightarrow 9(n-1) = 350 - 17 = 333$

$\Rightarrow n - 1 = 333 \div 9 = 37$

$\Rightarrow n = 38$

Now, $S_{38} = \frac{38}{2}(17+350) = 19 \times 367 = 6973$

7. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Solution: $a_{22} = 149 = a + 7 \times 21$

$\Rightarrow 149 - 147 = 2 = a$

Now, $S_{22} = \frac{22}{2}(2+149) = 11 \times 151 = 1661$

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Solution: Here d = 4 and a = 14 - 4 = 10

So, $a_{51} = 10 + 4 \times 50 = 210$

Now, $S_{51} = \frac{51}{2}(10+210) = 51 \times 110 = 5610$

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution: $S_7 = 49 = \frac{7}{2}(2a + 6d)$

$\Rightarrow 14a + 42d = 98$

$\Rightarrow a + 3d = 7$ ..... (4^{th} term).....(i)

$S_{17} = 289 = \frac{17}{2}(2a + 16d)$

$\Rightarrow 34a + 272d = 578$

$\Rightarrow a + 8d = 17$ .....(9^{th} term)

Deducting 4^{th} term from 9^{th} term we get

$5d = 10$

$\Rightarrow d = 2$

Putting the value of d in equation (i) we get a = 1

Now, $S_n = \frac{n}{2}(2 + 2(n-1)) = n(1 + n - 1) = n^2$

10. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth term.

Solution: First term = $4 \times 1 - 1^2 = 3$

$S_2 = 4 \times 2 - 2^2 = 4$

Second Term = 4 -3 = 1

So, d = -2

$a_3 = 3 - 2 \times 2 = -1$

$a_{10} = 3 - 2 \times 9 = -15$

$a_n = 3 - 2(n-1)$

11. Find the sum of the first 40 positive integers divisible by 6.

Solution: a = 6 , d = 6 and n = 40

$S_{40} = \frac{40}{2}(12 + 6 \times 39) = 20 \times 336 = 6720$

12. Find the sum of the first 15 multiples of 8.

Solution: a = 8, d = 8 and n = 15

$$S_{15} = \frac{15}{2}(16 + 8 \times 14) = 15 \times 64 = 960$$

13. Find the sum of the odd numbers between 0 and 50.

Solution: a = 1, d = 2, and n = 25

$$S_{25} = \frac{25}{2}(2 + 2 \times 24) = 625$$

14. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Solution: a = 200, d = 50 and n = 30

$$S_{30} = \frac{30}{2}(400 + 50 \times 29) = 27750$$

15. A sum of Rs. 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs. 20 less than its preceding prize, find the value of each of the prizes.

Solution: d = 20, n = 7 Sn = 700

$$S_7 = \frac{7}{2}(2a + 20 \times 6) = 700$$

$$\Rightarrow 7a + 420 = 700$$

$$\Rightarrow a = 280 \div 7 = 40$$

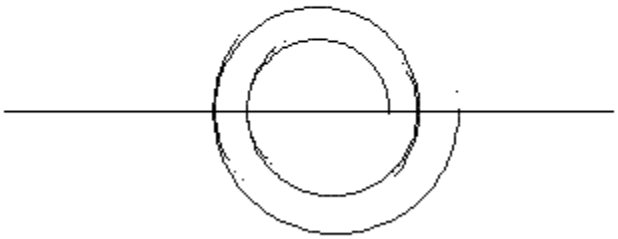
So, The AP: 40, 60, 80, 100, 120, 140, 160

16. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Solution: Number of Trees Planted = Class x 3
Class 1 = 3
Class 2 = 6
Class 3 = 9
It is clear that a = 3, d = 3 and n = 12

$$S_{12} = \frac{12}{2}(6+3\times11) = 234$$

17. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with cnetre A, fo radii 0.5 cm, 1 cm, 1.5 cm, 2 cm, …….. as shown in the figure. What is the total length of such spiral made up of thirteen consecutive semicircles?



Solution: Perimeter of first semi-circle = $\pi \times r = \frac{22}{7}\times\frac{1}{2} = \frac{11}{7}$

Perimeter of second semi-circle = $\frac{22}{7}\times1 = \frac{22}{7}$

So, d = $\frac{22}{7} - \frac{11}{7} = \frac{11}{7}$

$$S_{13} = \frac{13}{2}(2\times\frac{22}{7} + \frac{11}{7}\times12)$$

$$= \frac{13}{2}(\frac{44+132}{7}) = \frac{13}{2}\times\frac{176}{7} = 163.42 \text{ cm}$$

18. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

Solution: a = 20, d = -1

$$S_n = 200 = \frac{n}{2}(40 - 1(n-1))$$

$$\Rightarrow 400 = 41n - n^2$$

$$\Rightarrow n^2 + 41n + 400 = 0$$

$$\Rightarrow n^2 + 25n + 16n + 400 = 0$$

$$\Rightarrow n(n+25) + 16(n+25) = 0$$

$$\Rightarrow (n+16)(n+25) = 0$$

Let us check from both values of root
If number of rows is 16, then

$$a_{16} = 20 - 15 = 5$$

If number of rows is 25, then

$$a_{25} = 20 - 24 = -4$$

As negative value is not possible so there are 16 rows and there are 5 logs in the last row.

19. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

Solution: Distance Covered in picking the first potato = 5+5 = 10
Distance covered in picking the second potato = 2(5+3) = 16
Distance covered in picking the third potato = 2 (5+3+3) = 22
So, a = 10, d = 6 and n = 10

$$S_{10} = \frac{10}{2}(20 + 6 \times 9) = 74 \times 5 = 370 \text{ km}$$