

# JSUNIL TUTORIAL

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Solved

Assignments

**Q.1.** Based on Euclid's algorithm:  $a = bq + r$

Using Euclid's algorithm: Find the HCF of 825 and 175.

**Explanation:**

**Step 1.** Since  $825 > 175$ . Divide 825 by 175. We get, quotient = 4 and remainder = 125. This can be written as  $825 = 175 \times 4 + 125$

**Step II.** Now divide 175 by the remainder 125. We get quotient = 1 and remainder = 50. So we write  $175 = 125 \times 1 + 50$ .

**Step III.** Repeating the above step we now divide 125 by 50 and get quotient = 2 and remainder = 25. so  $125 = 50 \times 2 + 25$

**Step IV.** Now divide 50 by 25 to get quotient = 2 and remainder 0. Since remainder has become zero we stop here. Since divisor at this stage is 25, so the HCF of 825 and 175 is 25.

**Solution: This is how a student should write answer in his answer sheet:**

Since  $825 > 175$ , we apply division lemma to 825 and 175 to get

$$825 = 175 \times 4 + 125.$$

Since  $r \neq 0$ , we apply division lemma to 175 and 125 to get

$$175 = 125 \times 1 + 50$$

Again applying division lemma to 125 and 50 we get,

$$125 = 50 \times 2 + 25.$$

Once again applying division lemma to 50 and 25 we get.

$$50 = 25 \times 2 + 0.$$

Since remainder has now become 0, this implies that HCF of 825 and 125 is 25.

**Problems for practice;**

Find HCF of the following pairs using Euclid's division Lemma

(a) 34782 and 1892 (b) 588 and 240 (c) 80784 and 628

**Q.2.** Based on Showing that every positive integer is either of the given forms:

Solved example:

Prove that every odd positive integer is either of the form  $4q + 1$  or  $4q + 3$  for some integer  $q$ .

Explanation:

Euclid's division lemma  $a = bq + r$ .

Comparing this with the given integers  $(4q + 1)$  we get that  $b$  should be 4. If we divide any number by 4 possible remainders are 0, 1, 2 or 3 because fourth number will again be divided by 4. Ex  $12 \div 4$ ,  $r=0$ ;  $13 \div 4$ ,  $r=1$ ;  $14 \div 4$ ,  $r=2$ ;  $15 \div 4$ ,  $r=3$ ;  $16 \div 4$  once again  $r = 0$ . Hence possible remainders are 0, 1, 2 or 3. If  $r = 0$ , then we get  $a = 4q$ , If  $r = 1$  we get  $a = 4q + 1$  and so on till  $r = 3$  which will give  $a = 4q + 3$ . Since we want only odd integers our choices are  $4q + 1$  and  $4q + 3$ .

**Solution:**

Let  $a$  be any odd positive integer (first line of problem) and let  $b = 4$ . Using division Lemma we can write  $a = bq + r$ , for some integer  $q$ , where  $0 \leq r < 4$ . So  $a$  can be  $4q$ ,  $4q + 1$ ,  $4q + 2$  or  $4q + 3$ . But since  $a$  is odd,  $a$  cannot be  $4q$  or  $4q + 2$ . Therefore any odd integer is of the form  $4q + 1$  or  $4q + 3$ .

**Practice questions:**

Write the possible remainders when a number is divided by 5, 3, 7, 2,

Prove that every even positive integer is either of the form  $6q$ ,  $6q + 2$  or  $6q + 4$ .

Prove that every positive integer is either of the form  $3q$ ,  $3q + 1$  or  $3q + 2$  for some integer  $q$ .

**Q. 3.** Based on LCM and HCF:

Formula:  $LCM \times HCF = \text{product of numbers}$  Or  $\text{product of numbers} = LCM \times HCF$

**Hint:** If LCM or HCF is to be found then use the first formula. If value of any of the numbers is to found use second formula. That is, always keep the unknown variable on the LHS to avoid confusion.

**Solved example:**

Find HCF (26,91) if LCM(26,91) is 182

Sol: We know that  $LCM \times HCF = \text{Product of numbers}$ .

$$\text{or } 182 \times HCF = 26 \times 91$$

$$\text{or } HCF = \frac{26 \times 91}{182} = 13$$

Hence  $HCF(26, 91) = 13$ .

LCM and HCF of two numbers are 3024 and 6 respectively. If one of the number is 336 find the other number.

Sol: We know that

Product of numbers =  $LCM \times HCF$

Or Number =  $\frac{LCM \times HCF}{\text{Given number}}$

$$\text{Or number} = \frac{3024 \times 6}{336} = 54$$

Hence the other number is 54.

Solve similar questions from your text book.

**Q. 4.** Based on irrational numbers.

By heart the following jingles.

Let us assume on the contrary the  $\sqrt{c}$  is rational. That is we can find co-primes a and b ( $b \neq 0$ ) such that  $\sqrt{c} = a/b$ .

$c$  divides  $a^2$ . Hence it follows that  $c$  divides a.

So we can write  $a = c$ .

$c$  divides  $b^2$ . Hence it follows that  $c$  divides b.

Now a and b have at least  $c$  as a common factor.

But this contradicts the fact that a and b are co-primes.

This contradiction has arisen because of our incorrect assumption that  $\sqrt{c}$  is irrational.

Hence  $\sqrt{c}$  is rational.

**Sample question: prove that  $\sqrt{5}$  is irrational.**

**Solution:**

let us assume on the contrary that  $\sqrt{5}$  is rational. That is we can find co-primes a and b ( $b \neq 0$ ) such that  $\sqrt{5} = a/b$ . (1<sup>st</sup> jingle)

Or  $\sqrt{5}b = a$ .

Squaring both sides we get

$$5b^2 = a^2.$$

This means 5 divides  $a^2$ . Hence it follows that 5 divides a. (2<sup>nd</sup> jingle)

So we can write  $a = 5c$  for some integer c. (3<sup>rd</sup> jingle)

Putting this value of a we get

$$5b^2 = (5c)^2$$

$$\text{Or } 5b^2 = 25c^2$$

$$\text{Or } b^2 = 5c^2.$$

It follows that 5 divides  $b^2$ . Hence 5 divides b. (4<sup>th</sup> jingle)

Now a and b have at least 5 as a common factor. (5<sup>th</sup> jingle)

But this contradicts the fact that a and b are co-primes. (6<sup>th</sup> jingle)

This contradiction has arisen because of our incorrect assumption that  $\sqrt{5}$  is rational. (7<sup>th</sup> jingle)

Hence it follows that  $\sqrt{5}$  is irrational. (8<sup>th</sup> jingle)