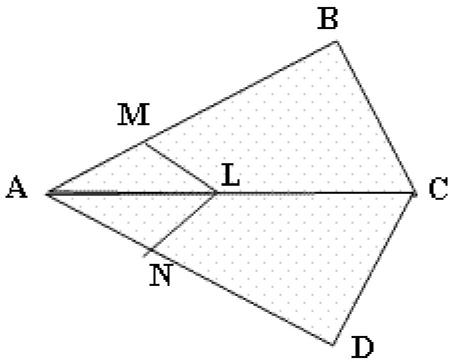
1. In the given figure, if LM || CB and LN || CD, prove that



$$\frac{AM}{AB} = \frac{AN}{AD}$$

Solution: In ΔABC & ΔAML

$\angle AML = \angle ABC$ (angles on the same side of transversal AB)

$\angle ALM = \angle ACB$ (angles on the same side of transversal AB)

$\angle BAC = \angle MAL$ (common angles)

So, $\Delta ABC \approx \Delta AML$

Similarly, $\Delta ADC \approx \Delta ANL$ can be proved

Now, In ΔABC

$\frac{AM}{AB} = \frac{AL}{AC}$ ....(i)

And In ΔADC

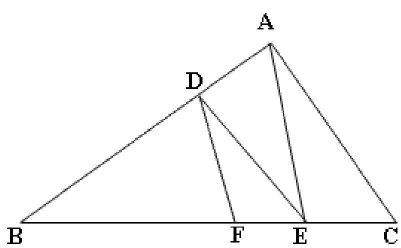
$\frac{AN}{AD} = \frac{AL}{AC}$ .... (ii)

From equations (i) and (ii) it is proved that

$$\frac{AM}{AB} = \frac{AN}{AD}$$

2. In the given figure, DE || AC and DF || AE. Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}$$



Solution: In ΔBEA

$\frac{BF}{FE} = \frac{BD}{DA}$ (because $\Delta BEA \approx \Delta BFD$) .... (i)

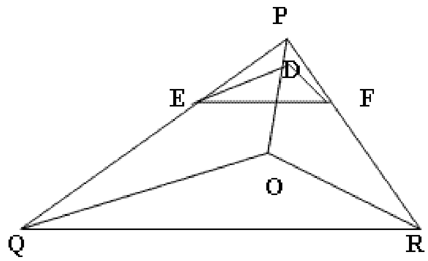
In ΔBCA

$\frac{BE}{EC} = \frac{BD}{DA}$ (because $\Delta BCA \approx \Delta BED$) .... (ii)

From equations (i) and (ii) it is proved that

$$\frac{BF}{FE} = \frac{BE}{EC}$$

3 . In the given figure, DE || OQ and DF || OR. Show that EF || QR.



Solution: As DE || OQ so $\Delta OPQ \approx \Delta DPE$

So, $\frac{QE}{PE} = \frac{OD}{DP}$ .... (i)

Similarly, As DF || OR so $\Delta ROP \approx \Delta FDP$

So, $\frac{RF}{FP} = \frac{OD}{DP}$ ....(ii)

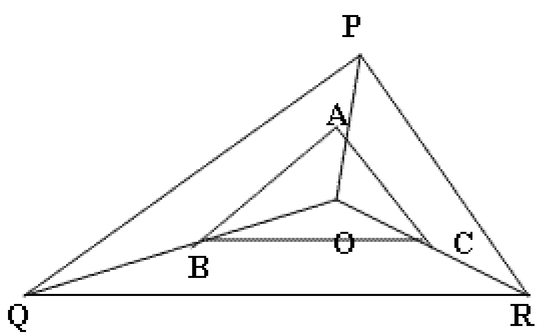
From equations (i) and (ii) it is clear that

$\frac{QE}{PE} = \frac{RF}{FP}$

Hence, $\Delta PQR \approx \Delta PEF$

So, QR || EF proved

6. In the given figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



Solution: As AB || PO

So, $\Delta POQ \approx \Delta AOB$

So, $\frac{PA}{AO} = \frac{QB}{BO}$ .... (i)

As AC || PR

So, $\Delta POR \approx \Delta AOC$

So, $\frac{PA}{AO} = \frac{RC}{CO}$ .... (ii)

From equations (i) and (ii) it is clear that

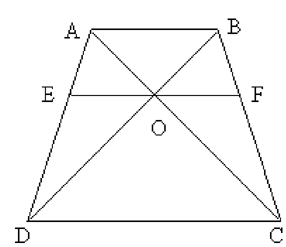
$\frac{QB}{BO} = \frac{RC}{CO}$

Hence, $\Delta QOR \approx \Delta BOC$

And, BC || QR proved

7. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that

$\frac{AO}{BO} = \frac{CO}{DO}$



Solution: Let us draw a line EF || AB and passing through O.
Now, it is clear that $\Delta ABD \approx \Delta EOD$

So, $\frac{AE}{AD} = \frac{BO}{OD}$ .... (i)

Similarly following can be proved

$\frac{BF}{FC} = \frac{AO}{OC}$ .... (ii)

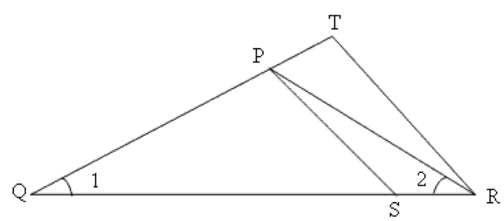
As diagonals of a trapezium are equal so,

$\frac{BO}{OD} = \frac{AO}{OC}$

Or, $\frac{AO}{BO} = \frac{CO}{DO}$ proved

8.

2. In the following figure $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \approx \Delta TQR$.



Solution: In ΔPQR

$\angle 1 = \angle 2$

So, QP = PR (isosceles triangle)

So, $\frac{QR}{QS} = \frac{QT}{PR} = \frac{QT}{QP}$

Hence, PS || TR

$\Delta PQS \approx \Delta TQR$ proved

9.

3. S and T are points on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Solution: In ΔRPQ and ΔRTS

$\angle RPQ = \angle RTS$ (given)

$\angle R = \angle R$ (common angle)

So, $\angle RQP = \angle RST$ (remaining angle of the triangle)

Hence, ΔRPQ ~ ΔRTS. proved

10 . In the following figure, if ΔABE Δ ACD, show that Δ ADE ~ Δ ABC

Solution: In ΔABE and ΔACD

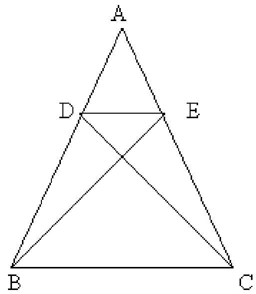
AB = AC

AD = AE

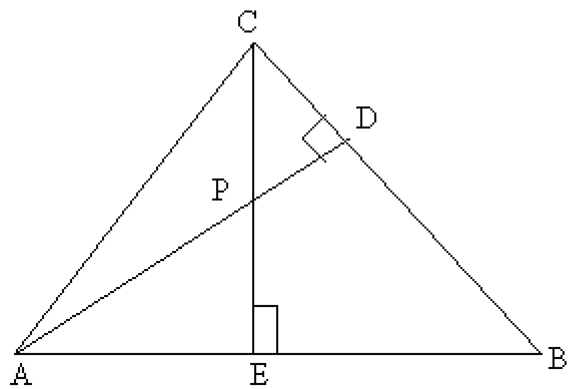
So, $\frac{AB}{AD} = \frac{AC}{AE}$

Hence, DE || BC

And, Δ ADE ~ Δ ABC proved



11. In the following figure, altitudes AD and CE of ABC intersect each other at the point P. Show that:



(i) **ΔAEP ~ ΔCDP**

Solution: In ΔAEP and ΔCDP

$\angle AEP = \angle CDP$

$\angle APE = \angle CPD$ (opposite angles)

So the third angle will automatically be equal

Hence, ΔAEP ~ ΔCDP is proved.

(ii) ΔABD ~ ΔCBE

Solution: $\angle ADB = \angle CEB$

$\angle ABD = \angle CBE$ (common angles)

So, ΔABD ~ ΔCBE is proved

(iii) ΔAEP ~ ΔADB

Solution: $\angle AEP = \angle ADB$

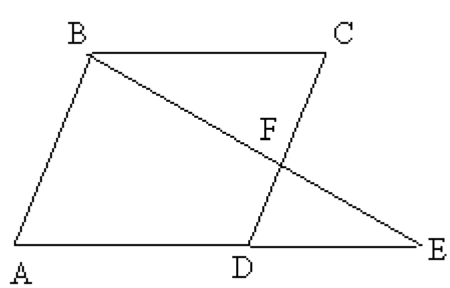
$\angle EAP = \angle BAD$ (common angles)

(iv) ΔPDC ~ ΔBEC

Solution: $\angle PDC = \angle BEC$

$\angle DCP = \angle BCE$ (common angles)

12 E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.



Solution: $\angle CBF = \angle AEB$ (alternate angles)

$\angle BCF = \angle CDE$ (alternate angles)

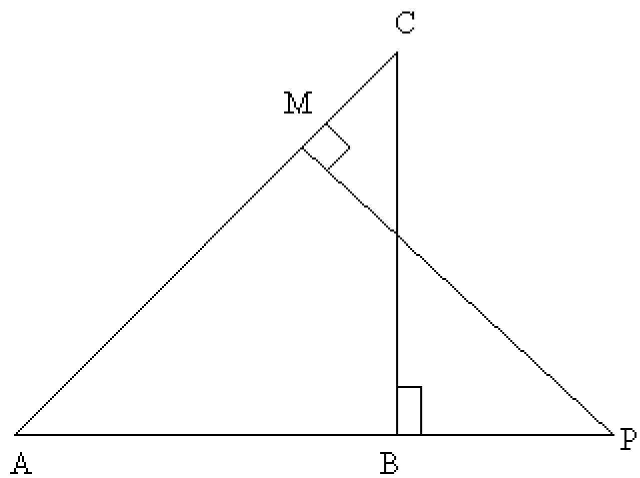
$\angle CDE = \angle BAE$ (alternate angles)

So, $\angle BCF = \angle BAE$

So, ΔABE ~ ΔCFB. Proved

13. In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

(i) ΔABC ~ ΔAMP



(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Solution: In ΔCMO and PBO

$\angle CMO = \angle PBO$

$\angle COM = \angle POB$ (opposite angles)

So, $\angle C = \angle P$

Now, $\angle ABC = \angle AMP$

And $\angle C = \angle P$

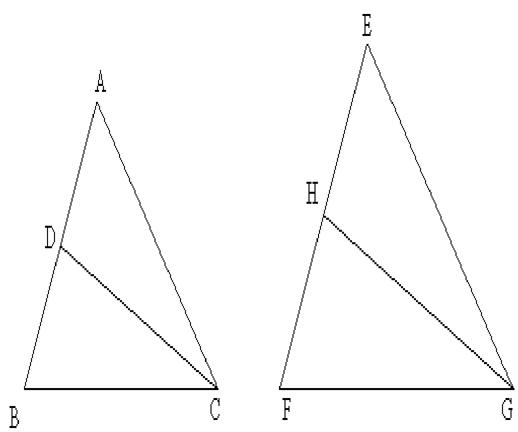
So, ΔABC ~ ΔAMP is proved

As per right angled triangle similarity theorem hypote
sides will be in the same ratio.

So, $\frac{CA}{PA} = \frac{BC}{MP}$ is proved

14. CD and GH are respectively the bisectors of angle ACB and angle EGF such that D and H lie on sides AB and EF of ΔABC and ΔEFG respectively.

If ΔABC ~ΔEFG, show that:



(i) $\frac{CD}{GH} = \frac{AC}{EG}$

Solution: In ΔADC and Δ EHG

$\angle DAC = \angle HEG$ (corresponding angles of similar triangles)

$\angle ACD = \angle EGH$ (half of corresponding angles)

So, ΔADC~ΔEHG

Hence, $\frac{CD}{GH} = \frac{AC}{EG}$ proved

(ii) ΔDCB ~ ΔHGF

Solution: $\angle DBC = \angle HFG$ (corresponding angles of similar triangles)

$\angle BCD = \angle FGH$ (half of corresponding angles)

Hence, ΔDCB ~ ΔHGF proved

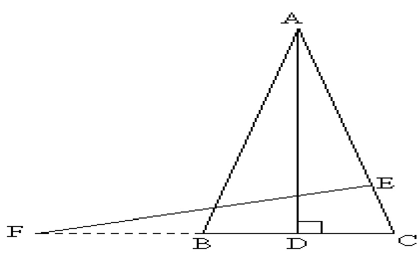
(iii) ΔDCA ~ ΔHGE

Solution: $\angle ACD = \angle EGH$ (half of corresponding angles)
$\angle DAC = \angle HEG$ (corresponding angles of similar triangles)
Hence, ΔDCA ~ ΔHGE proved

15.
9. . In the given figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥BC and EF ⊥? AC, prove that ΔABD ~ ΔECF.

Solution: $\angle ADB = \angle FEC$
$\angle ABD = \angle FCE$ (angles of equal sides of an isosceles triangle)
So, ΔABD ~ ΔECF. proved

by A A similarity

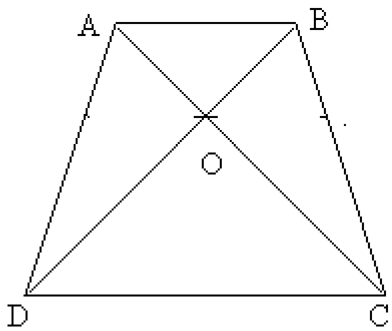


16.
11. . D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that CA^2 = CB.CD.

Solution: In ΔBAC and ΔADC
$\angle ADC = \angle BAC$
$\angle ACD = \angle ACB$
So ΔBAC ~ΔADC
$$\frac{CA}{CB} = \frac{CD}{CA}$$
Or, $CA^2 = CB \times CD$ proved

17 . In the given figure of trapezium, ABD and DBC are two triangles on the same base BD. If AC intersects BD at O, show that

$$\frac{ar(ABD)}{ar(DBC)} = \frac{AO}{CO}$$



Solution: As it is an isosceles trapezium, so in ΔABD and ΔCDB

ΔABO ~ ΔCDO can be proved as follows:

$\angle AOB = \angle COD$ (opposite angles)

$\angle ABO = \angle CDO$ (alternate angles)

ΔAOD ~ΔBOC can be proved as follows:

$\angle AOD = \angle COB$

AC = BD (diagonals are equal in isosceles trapezium)

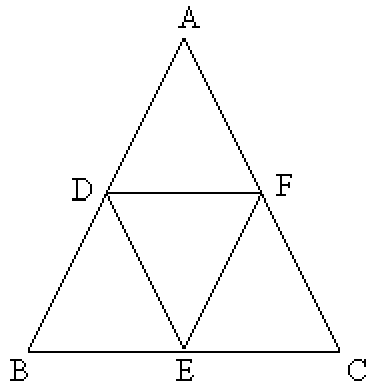
So, it is clear that

ΔAOD + ΔAOB ~ ΔBOC + ΔCOD

Or, ΔABD ~ ΔCDB

So, $\frac{ar(ABD)}{ar(DBC)} = \frac{AO}{CO}$ is proved

18. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.



Solution: ΔABC ~ΔADF can be proved as follows:

$\angle A = \angle A$

$\angle AFD = \angle ACB$ (angles on the same side of transversal)

$\angle ADF = \angle ABC$ (angles on the same side of transversal)

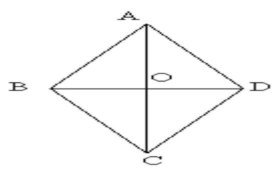
Similarly, ΔBDE, ΔECF and Δ DEF can be proved to be similar to the bigger triangle.

So, $\Delta ADF \cong \Delta DBE \cong FEC \cong EFD$

Ratio of areas of smaller triangle to that of bigger triangle

= 4 : 1

19 . Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.



Solution: All four triangles will be congruent

In ΔAOD

$AD^2 = AO^2 + OD^2$

Similarly, $AB^2 = AO^2 + BO^2$

$CB^2 = CO^2 + BO^2$

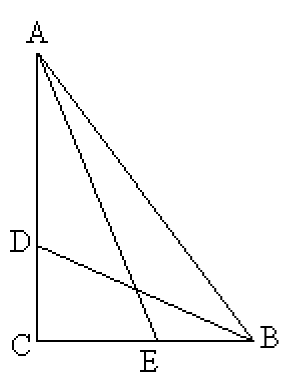
$CD^2 = CO^2 + OD^2$

Adding all we get

$AD^2 + AB^2 + BC^2 + CD^2 = AC^2 + BD^2$

20. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C.

Prove that $AE^2 + BD^2 = AB^2 + DE^2$.



Solution: In ΔACE

$AE^2 = AC^2 + CE^2$

In ΔDCB

$BD^2 = DC^2 + CB^2$

In ΔACB

$AB^2 = AC^2 + CB^2$

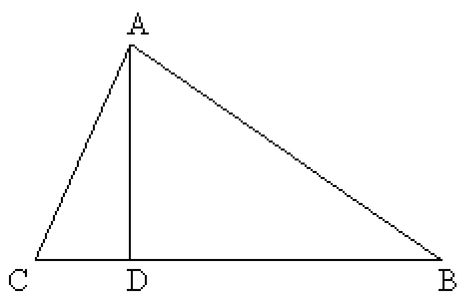
Adding first two we get

$AE^2 + BD^2 = AC^2 + CE^2 + DC^2 + CB^2$

$AE^2 + BD^2 = AB^2 + DE^2$

Proved

21. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3 CD. Prove that $2\ AB^2 = 2\ AC^2 + BC^2$.



Solution: $AB^2 = AD^2 + DB^2$ ………….. (i)

$AC^2 = AD^2 + CD^2$ ……………………. (ii)

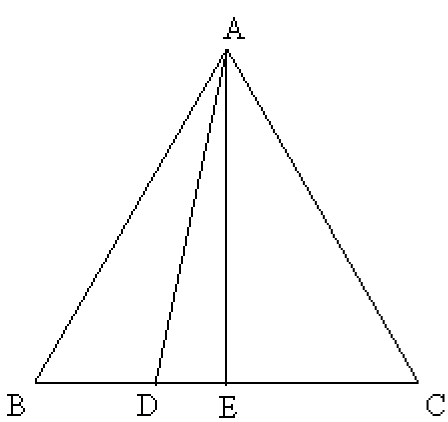
$2AB^2 = 2AD^2 + 2DB^2 = 2AD^2 + 18CD^2$

$= AD^2 + CD^2 + AD^2 + CD^2 + 16\ CD^2$

$= 2AC^2 + 16CD^2$ (as per equation (ii))

$= 2AC^2 + BC^2$ (BC = 4CD)

22. In an equilateral triangle ABC, D is a point on side BC such that BD = 1/3 BC. Prove that $9\ AD^2 = 7\ AB^2$.



Solution: $AD = \frac{a\sqrt{3}}{2}$

$DE = \frac{a}{6}$

In ΔABE

$AD^2 = AE^2 + DE^2$

$AD^2 = \frac{3a^2}{4} + \frac{a^2}{36} = \frac{28a^2}{36} = \frac{7a^2}{9}$

$\Rightarrow 9AD^2 = 7a^2 = 7AB^2$