Use of Factor Theorem IX

Division Algorithm for Polynomials:

If a polynomial f(x) is divided by a non-zero polynomial g (x) then there exist unique polynomials q (x) and r (x) such that f(x)=g(x)q(x) + r(x) where either r (x) = 0 or deg r (x) < deg g(x).Here dividend = f(x), divisor = g (x), quotient = q (x) and remainder = r (x). Thus if g (x) is a quadratic polynomial, then remainder r (x) is of the form ax+b, where a, b may be zero. If divisor g (x) is a linear polynomial then r (x) is a constant polynomial, i.e., r (x) = c, where c may be zero.

1. A non-zero polynomial g (x) is called a factor of any polynomial f (x) iff there exists some polynomial q (x) such that f(x) = g(x)q(x), i.e. iff on dividing f(x) by g (x), the remainder is zero.
2. **Remainder Theorem:**
   If a polynomial f(x) is divided by (x -a), then remainder = f(a).
   If a polynomial f(x) is divided by (x +a), then remainder = f(-a).
   If a polynomial f(x) is divided by (ax +b), then remainder = f(-b/a).
3. **Factor Theorem:**
   If f(x) is a polynomial and a is a real number, then (x -a) is a factor of f(x) iff f(a)=0.

Exercise

1. Divide $2x^3 - 7x^2 + 5x + 9$ by $2x - 3$ by long division method. Mention the dividend, divisor, quotient and remainder.
2. Using remainder theorem, find the remainder when $2x^3 - 7x^2 + 5x + 9$ is divided by $2x - 3$.
3. Find the remainder (without division) on dividing f(x) by x +3, where
   (i) $f(x) = 2x^2 - 7x - 1$ (ii) $f(x) = 3x^3 - 7x^2 + 11x + 1$.
4. Let $f(x) = 2x^2 - 7x - 1$. Find the remainder when f(x) is divided by
   (i) x -3 (ii) 2x -3 (iii) x /2 -3 (iv) 2(x -3)
   (v) k (x -3), $k \neq 0$ (vi) x (vii) 4 x
5. Let $f(x) = 3x^3 - 7x^2 + 1$. Find the remainder when f(x) is divided by
   (i) x +2 (ii) 2 x +2 (iii) 1(x +2)/2 (iv) 3(x +2)
   (v) k (x +2), $k \neq 0$ (vi) x (vii) 10 x
6. Using remainder theorem, find the value of a if the division of $x^3 + 5x^2 - ax + 6$ by (x -1) leaves the remainder 2 a.
7. Show that x -1 is a factor of $2x^2 + x - 3$. Hence factorise $2x^2 + x - 3$ completely.
8. Show that 2 x +3 is a factor of $6x^2 + 5x - 6$. Hence find the other factor.
9. Show that x +2 is a factor of $f(x) = x^3 + 2x^2 - x - 2$. Hence factorise f(x) completely.
10. Show that x -1 is a factor of $x^5 - 1$ while $x^5 + 1$ is not divisible by x -1.
11. Find the constant k if 2 x -1 is a factor of $f(x) = 4x^2 + kx + 1$. Using this value of k, factorise f(x) completely.
12. The expression $2x^3 + ax^2 + bx - 2$ leaves remainders of 7 and 0 when divided by 2 x -3 and x +2 respectively. Calculate the values of a and b. With these values of a and b, factorise the expression completely.
13. If x +1 and x -1 are factors of $f(x) = x^3 + 2ax + b$, calculate the values of a and b. Using these values of a and b, factorise f(x) completely.
14. If $x^2 - 1$ is a factor of $f(x) = x^4 + ax + b$, calculate the values of a and b. Using these values of a and b, factorise f(x).
15. Given that $x^2 - x - 2$ is a factor of $x^3 + 3x^2 + ax + b$, calculate the values of a and b and hence find the remaining factor.
16. The polynomial $x^4 + bx^3 + 59x^2 + cx + 60$ is exactly divisible by $x^2 + 4x + 3$. Find the values of b and c.

Answers

1. Dividend = $2x^3 - 7x^2 + 5x + 9$, divisor = $2x - 3$
quotient = $x^2 - 2x - 1/2$, remainder = $-21/2$

2. -21/2 **3.** (i) 38 (ii) -176

4. (i) -4 (ii) - 7 (iii) 29 (iv) -4 (v) -4 (vi) -1 (vii) -1

5. (i) -51 (ii) -9 (iii) -51 (iv) -51 (v) -51 (vi) 1 (vii) 1

6. 4 **7.** $(x-1)(2x+3)$ **8.** $3x-2$

9. $(x-1)(x+1)(x+2)$ **11.** $k = -4$; $(2x-1)^2$

12. $a = 3$, $b = -3$; $(x+2)(2x+1)(x-1)$

13. $a = -1/2$, $b = 0$, $x(x-1)(x+1)$

14. $a = 0$, $b = -1$, $(x-1)(x+1)(x^2+1)$

15. $a = -6$, $b = -8$, third factor is $x + 4$

16. $b = 13$, $c = 107$